Triangular Arbitrage

Cross-market pricing error

Concept

Triangular arbitrage exploits inconsistencies between three exchange rates or trading pairs that share a common set of assets. Starting from a base asset, you convert through a cycle of markets and return to the base. If the product of the rates around the cycle differs from 1, a risk-free profit exists (ignoring fees). On a single exchange this is the classic $A \to B \to C \to A$ loop.

Setup and Notation

Let r_{XY} be the amount of asset Y received per unit of asset X (the price of X quoted in Y). For the cycle $A \to B \to C \to A$, one unit of A becomes

$$Q = r_{AB}\, r_{BC}\, r_{CA}.$$

Define the **cycle product** $\Pi = r_{AB}\, r_{BC}\, r_{CA}$.

No-Arbitrage Condition

In a frictionless, perfectly consistent market,

$$r_{AB}\, r_{BC}\, r_{CA} = 1, \qquad \text{equivalently} \qquad r_{XY} = \frac{1}{r_{YX}}.$$

Any deviation is a cross-market pricing error.

Profit Formulas

The gross return of one full loop, per unit of base asset A, is

$$\boxed{\Pi = r_{AB}\, r_{BC}\, r_{CA}} \qquad \text{Net profit rate} = \Pi - 1.$$

A profitable opportunity (long cycle) requires $\Pi > 1$; the reverse cycle $A \to C \to B \to A$ is profitable when $\Pi < 1$, with product $1/\Pi$.

Including transaction costs

Let f be the proportional fee per trade (e.g. $f = 0.001$ for 0.1%). With three trades, the net multiplier is

$$\Pi_{\text{net}} = \Pi\,(1-f)^3, \qquad \text{profitable} \iff \Pi\,(1-f)^3 > 1.$$

More generally, for an n-leg cycle with per-leg fees f_i,

$$\Pi_{\text{net}} = \left(\prod_{i=1}^{n} r_i\right) \prod_{i=1}^{n} (1-f_i).$$

Order-book (executable) version

Quoted mid-prices overstate profit. Using the executable side of each book—you *buy at the ask* and *sell at the bid*—replace each r with the fill-side rate:

$$\Pi_{\text{exec}} = \frac{1}{a_{AB}} \cdot \frac{1}{a_{BC}} \cdot b_{CA},$$

where a denotes ask and b denotes bid for the relevant leg's direction. Only $\Pi_{\text{exec}}(1-f)^3 > 1$ is a genuine opportunity.

Worked Example

Suppose on one exchange:

$$r_{\text{USDT}\to\text{BTC}} = \tfrac{1}{64000}, \quad r_{\text{BTC}\to\text{ETH}} = 34.7, \quad r_{\text{ETH}\to\text{USDT}} = 1850.$$

Then

$$\Pi = \frac{34.7 \times 1850}{64000} = \frac{64195}{64000} \approx 1.00305.$$

Gross edge $\approx 0.305\%$. With $f = 0.1\%$ per leg, $\Pi_{\text{net}} = 1.00305 \times 0.999^3 \approx 1.00005$, a thin but positive $\approx 0.005\%$.

Practical Notes

- Edge is fleeting: latency and slippage erode Π_{exec} within milliseconds.
- Available *depth* caps trade size; the top-of-book rate holds only for small volume.
- Withdrawal/transfer times matter for cross-*exchange* triangles; single-exchange loops avoid this.

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