Equity Market Arbitrage (TASI)

ETF NAV arbitrage & statistical pairs trading

Concept

Two distinct inefficiencies arise in a cash equity market such as the Saudi Exchange (TASI). **ETF arbitrage** is a true, near-riskless arbitrage: an ETF share is a claim on a known basket, so its market price must track the basket's net asset value; the creation/redemption mechanism enforces this. **Pairs trading** is a *statistical* arbitrage: two economically related stocks share a long-run equilibrium, and temporary divergences are expected—but not guaranteed—to revert. The first is enforced by a contractual mechanism, the second only by a statistical relationship.

Part I — ETF NAV Arbitrage

Net asset value

For an ETF holding n constituents with w_i shares of stock i (price P_i), cash C, liabilities L, and N_{sh} shares outstanding:

$$\boxed{\text{NAV} \;=\; \frac{\sum_{i=1}^{n} w_i P_i + C - L}{N_{\text{sh}}}}$$

The intraday estimate published during the session is the *iNAV* (indicative NAV), recomputed from live constituent prices.

Premium / discount

Let E be the ETF's traded market price. The deviation is

$$\delta \;=\; \frac{E - \text{NAV}}{\text{NAV}} \times 100\%,$$

$\delta > 0$ a **premium**, $\delta < 0$ a **discount**. The no-arbitrage band is

$$|E - \text{NAV}| \;\le\; c,$$

where c aggregates creation/redemption fees, commissions, bid–ask spreads on the ETF and all constituents, and settlement financing.

The two trades

Condition	Trade	Mechanism
$E > \text{NAV} + c$ (premium)	Buy basket, sell ETF	Create new ETF units, deliver
$E < \text{NAV} - c$ (discount)	Buy ETF, sell basket	Redeem ETF units for basket

Profit per share, net of costs:

$$\pi \;=\; |E - \text{NAV}| - c.$$

Only an Authorised Participant can create/redeem directly; other participants trade the convergence without the mechanism, which reintroduces risk.

Tracking difference

Over a holding period, ETF return should match index return less fees:

$$\text{Tracking difference} = R_{\text{ETF}} - R_{\text{index}}, \qquad \text{Tracking error} = \text{sd}\big(R_{\text{ETF}} - R_{\text{index}}\big).$$

Persistent negative tracking difference beyond the expense ratio signals structural drag, not an arbitrage.

Worked example

Basket value per share = 42.50, ETF trades at $E = 43.10$, total round-trip cost $c = 0.25$. Then $\delta = (43.10 - 42.50)/42.50 = 1.41\%$ premium and $\pi = 0.60 - 0.25 = 0.35$ per share. Buy the basket, sell the ETF, create units to close.

Part II — Statistical Pairs Trading

The spread

For two related stocks with prices P_A, P_B, define the spread using log prices and a hedge ratio β:

$$S_t \;=\; \ln P_{A,t} \;-\; \beta \ln P_{B,t}.$$

β is estimated by ordinary least squares,

$$\ln P_{A,t} = \alpha + \beta \ln P_{B,t} + \varepsilon_t, \qquad \hat{\beta} = \frac{\mathrm{Cov}(\ln P_A, \ln P_B)}{\mathrm{Var}(\ln P_B)}.$$

The pair is tradable only if S_t is **stationary** (cointegrated): the residual ε_t must not wander. This is tested with an Engle–Granger or Augmented Dickey–Fuller test, rejecting a unit root at the chosen confidence level.

Normalised divergence (z-score)

Standardise the spread over a rolling window of length m:

$$\boxed{z_t \;=\; \frac{S_t - \mu_t}{\sigma_t}} \qquad \mu_t = \frac{1}{m}\sum_{k=1}^{m} S_{t-k}, \qquad \sigma_t = \sqrt{\frac{1}{m-1}\sum_{k=1}^{m}\big(S_{t-k} - \mu_t\big)^2}.$$

Trading rule

Signal	Position	Interpretation
$z_t \geq +z_{\text{in}}$	Short A, long β units of B	A rich relative to B
$z_t \leq -z_{\text{in}}$	Long A, short β units of B	A cheap relative to B
$\lvert z_t \rvert \leq z_{\text{exit}}$	Close	Reverted to equilibrium
$\lvert z_t \rvert \geq z_{\text{stop}}$	Stop out	Relationship may have broken

Common choices are $z_{\text{in}} = 2$, $z_{\text{exit}} = 0$, $z_{\text{stop}} = 3$–4.

Mean reversion speed

Model the spread as an Ornstein–Uhlenbeck process,

$$dS_t \;=\; \theta\,(\mu - S_t)\,dt \;+\; \sigma\,dW_t,$$

with reversion rate $\theta > 0$. The expected time for a deviation to halve is the **half-life**

$$t_{1/2} \;=\; \frac{\ln 2}{\theta},$$

where θ is estimated from the regression $\Delta S_t = a + b\,S_{t-1} + u_t$ via $\theta = -\ln(1+b)$. Pairs with very long half-lives tie up capital; very short ones are consumed by costs.

Position sizing and P&L

Dollar-neutral sizing takes β units of B per unit of A. Gross P&L on a reverting round trip is approximately

$$\pi \;\approx\; \lvert z_{\text{entry}} - z_{\text{exit}} \rvert \cdot \sigma_t \cdot V,$$

for notional V, less financing, borrow cost on the short leg, and two round-trip spreads.

Worked example

A pair has $\hat{\beta} = 0.85$, rolling $\mu = 0.42$, $\sigma = 0.031$, and current $S_t = 0.487$. Then

$$z_t = \frac{0.487 - 0.42}{0.031} = 2.16 \ \geq \ 2,$$

so short A and long 0.85 units of B per unit of A, targeting exit at $z = 0$.

Key Distinction

ETF arbitrage is enforced by creation/redemption: convergence is contractual and the trade is genuinely riskless within the cost band. Pairs trading relies on a historical statistical relationship that can break permanently—through a merger, regulatory change, or shift in business model—so it requires stop losses, position limits, and periodic re-testing of cointegration. It is *relative value*, not arbitrage in the strict sense.

Practical Notes

- Saudi market constraints matter: settlement cycle, daily price fluctuation limits, foreign ownership rules, and the availability and cost of stock borrow for short legs.
- Short selling must be via an approved regulated route; where borrow is unavailable, pairs trades reduce to long-only tilts.
- Re-estimate β and re-test cointegration on a rolling basis; a stale hedge ratio silently converts a neutral book into a directional one.
- Screen for liquidity first—wide spreads on either leg erase a 2σ edge.