Spot / Future Arbitrage

Cash-and-carry opportunities

Concept

A futures (or perpetual) contract and its underlying spot asset must satisfy a *cost-of-carry* relationship. When the futures price strays from fair value, you lock in a risk-free spread by taking offsetting positions: buy the cheap leg, sell the rich leg, and hold to convergence at expiry. **Cash-and-carry** = buy spot, sell future (future rich); **reverse cash-and-carry** = sell spot short, buy future (future cheap).

Fair Value: Cost of Carry

Let S_0 be the spot price, T the time to expiry (years), r the risk-free rate, q the yield/convenience income of holding the asset (dividends, staking), and u the storage cost rate. The no-arbitrage forward price is

$$\boxed{F_0 \;=\; S_0\, e^{(r-q+u)\,T}} \qquad \text{(continuous compounding).}$$

With discrete/simple rates over the period:

$$F_0 \;=\; S_0\,\big(1 + (r-q+u)\,T\big).$$

The quantity $F_0 - S_0$ is the *basis*; $b = (r - q + u)$ is the net carry rate.

Arbitrage Conditions

- **Cash-and-carry** (future overpriced): if $F_{\text{mkt}} > S_0\, e^{(r-q+u)T}$, buy spot financed at r, sell the future. Profit at expiry per unit:
$$\pi \;=\; F_{\text{mkt}} - S_0\, e^{(r-q+u)T}.$$
- **Reverse cash-and-carry** (future underpriced): if $F_{\text{mkt}} < S_0\, e^{(r-q+u)T}$, short spot (invest proceeds at r), buy the future. Profit per unit:
$$\pi \;=\; S_0\, e^{(r-q+u)T} - F_{\text{mkt}}.$$

Including round-trip transaction cost c, the trade is profitable only if $|F_{\text{mkt}} - F_0| > c$.

Implied Rate / Annualized Basis

Traders often quote the opportunity as an *annualized* return. Solving the fair-value relation for the implied carry:

$$r_{\text{impl}} \;=\; \frac{1}{T}\ln\!\left(\frac{F_{\text{mkt}}}{S_0}\right) + q - u, \qquad \text{Annualized basis} \;=\; \left(\frac{F_{\text{mkt}}}{S_0} - 1\right)\frac{1}{T}.$$

The edge over funding is $r_{\text{impl}} - r$.

Perpetual Futures (Funding-Rate Variant)

Perpetuals have no expiry; convergence is enforced by a periodic **funding payment** g exchanged between longs and shorts. A delta-neutral carry (long spot, short perpetual) earns the funding stream:

$$\text{Return per interval} \;=\; g, \qquad \text{APR} \;=\; g \times (\text{intervals per year}),$$

e.g. g paid every 8h gives $3 \times 365 = 1095$ intervals. Positive g pays the short; the position is (near) market-neutral, earning funding minus fees and slippage.

Worked Example

$S_0 = 64{,}000$, quarterly future ($T = 0.25$), $r = 5\%$, $q = u = 0$:

$$F_0 = 64000\, e^{0.05\times 0.25} = 64000 \times 1.01258 = 64{,}805.$$

If the market quotes $F_{\text{mkt}} = 65{,}500$, the cash-and-carry edge is $65500 - 64805 = 695$ per unit, i.e. an implied rate $r_{\text{impl}} = \frac{1}{0.25}\ln(65500/64000) = 9.26\%$, a 4.26% spread over funding.

Practical Notes

- Convergence is guaranteed at expiry (dated futures) but path P&L swings via margin.
- Reverse carry needs borrowable spot; borrow fees enter u.
- Funding on perps is variable—model it as a stochastic income stream, not a constant.

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