Options: Put–Call Parity

The fundamental no-arbitrage relation

Concept

Put–call parity ties together the prices of a European call and put with the same strike K and expiry T on the same underlying. A portfolio replicating the stock can be built from options plus a bond, so their prices are locked into a rigid relation. Any violation is a pure arbitrage: you buy the cheap synthetic and sell the rich one.

The Parity Relation

For European options on a non-dividend-paying asset with spot S_0, risk-free rate r:

$$\boxed{C - P \;=\; S_0 - Ke^{-rT}}$$

Equivalently $C + Ke^{-rT} = P + S_0$ (a call plus cash equals a put plus stock; the "fiduciary call = protective put" identity).

With dividends / carry

If the asset pays a continuous yield q (or present-value dividends D):

$$C - P \;=\; S_0e^{-qT} - Ke^{-rT}, \qquad \text{or} \qquad C - P \;=\; S_0 - D - Ke^{-rT}.$$

Using the forward $F_0 = S_0e^{(r-q)T}$, parity is compactly $C - P = e^{-rT}(F_0 - K)$.

Derivation (Replication)

Consider two portfolios held to expiry:

A one call (C) + cash Ke^{-rT} (grows to K).
B one put (P) + one share (S_0).

At expiry both are worth $\max(S_T, K)$ regardless of S_T:

$$\underbrace{\max(S_T - K, 0) + K}_{\text{A}} = \max(S_T, K) = \underbrace{\max(K - S_T, 0) + S_T}_{\text{B}}.$$

Equal payoffs in every state $\Rightarrow$ equal prices today $\Rightarrow C + Ke^{-rT} = P + S_0$.

Arbitrage from a Violation

Define the parity gap

$$\Delta \;=\; \big(C - P\big) - \big(S_0 - Ke^{-rT}\big).$$

Case $\Delta > 0$ (call rich / put cheap) — "short the combo"

The call side is overpriced relative to the put side. Trade:

- Sell call, buy put, buy stock, borrow Ke^{-rT}.

Leg	Cash flow t=0	Payoff $S_T < K$	Payoff $S_T \geq K$
Sell call	$+C$	0	$-(S_T - K)$
Buy put	$-P$	$K - S_T$	0
Buy stock	$-S_0$	S_T	S_T
Borrow Ke^{-rT}	$+Ke^{-rT}$	$-K$	$-K$
Net	$\boxed{+\Delta}$	0	0

You collect $\Delta > 0$ today with zero payoff at expiry: risk-free profit.

Case $\Delta < 0$ (put rich / call cheap) — "long the combo"

Reverse every leg: buy call, sell put, short stock, lend Ke^{-rT}. Net inflow today is $-\Delta > 0$ with zero terminal payoff.

American Options (Inequality Bounds)

Early exercise breaks the equality; parity becomes a two-sided bound. For American options on a non-dividend payer:

$$S_0 - K \;\leq\; C_{\text{am}} - P_{\text{am}} \;\leq\; S_0 - Ke^{-rT}.$$

An observed $C - P$ outside this band signals arbitrage even for American-style contracts.

Worked Example

$S_0 = 100$, $K = 100$, $r = 5\%$, $T = 1$, no dividends. Fair $C - P = 100 - 100e^{-0.05} = 4.88$. If the market shows $C = 9.00$, $P = 3.50$, then $C - P = 5.50$ and $\Delta = 5.50 - 4.88 = 0.62$. Execute the "short the combo" trade to bank \$0.62 per share, risk-free.

Practical Notes

- Use *executable* bid/ask, not mid: cross the spread on four legs.
- Match strike *and* expiry exactly; borrow cost for shorting stock enters r.
- Discrete dividends must be modeled precisely—mis-estimating D fakes a false gap.