Option Chain Analysis

Convexity, butterfly & calendar arbitrage, pricing bounds

Concept

An arbitrage-free option chain must obey *shape* constraints across strikes and expiries. Call price as a function of strike must be positive, decreasing, and *convex*; across maturities it must be non-decreasing (calendar). Violations of these static, model-free conditions are pure arbitrages—no volatility model or forecast required. We use these to scan a whole chain for inconsistencies.

Notation: $C(K)$, $P(K)$ are call/put prices at strike K, all same expiry T; $C(K,T)$ adds the maturity. Discount factor $DF = e^{-rT}$, forward $F = S_0 e^{(r-q)T}$.

1. Monotonicity Bounds (Vertical)

Calls fall, puts rise in strike; slopes are bounded by the discount factor:

$$-DF \;\le\; \frac{\partial C}{\partial K} \;\le\; 0, \qquad 0 \;\le\; \frac{\partial P}{\partial K} \;\le\; DF.$$

Discretely, for adjacent strikes $K_1 < K_2$:

$$0 \;\le\; C(K_1) - C(K_2) \;\le\; (K_2 - K_1)\, DF.$$

Vertical-spread arbitrage: if $C(K_1) - C(K_2) < 0$, buy $C(K_1)$/sell $C(K_2)$ for a credit with non-negative payoff. If $C(K_1) - C(K_2) > (K_2 - K_1)DF$, do the reverse.

2. Convexity / Butterfly Arbitrage

For three equally spaced strikes $K_1 < K_2 < K_3$ with $K_2 = \frac{K_1+K_3}{2}$, convexity of $C(K)$ requires the **butterfly** to have non-negative cost:

$$\boxed{C(K_1) - 2\,C(K_2) + C(K_3) \;\ge\; 0}$$

(The same holds for puts.) The butterfly $+1\,C(K_1) - 2\,C(K_2) + 1\,C(K_3)$ has a payoff that is ≥ 0 everywhere (a "tent" peaking at K_2), so its price cannot be negative.

Arbitrage

If $C(K_1) - 2C(K_2) + C(K_3) < 0$, **buy the butterfly** (long the wings, short 2 body): you receive a credit today and a non-negative payoff at expiry—risk-free. General unequal spacing convexity condition:

$$C(K_2) \;\le\; \lambda\, C(K_1) + (1-\lambda)\, C(K_3), \qquad \lambda = \frac{K_3 - K_2}{K_3 - K_1}.$$

Violation $\Rightarrow$ buy the (weighted) butterfly $\lambda\, C(K_1) + (1-\lambda)C(K_3) - C(K_2)$.

Worked butterfly example

Strikes $95, 100, 105$; calls $8.00, 5.00, 2.90$. Then $8.00 - 2(5.00) + 2.90 = -0.10 < 0$. Buy 1×95, sell 2×100, buy 1×105: collect \$0.10 now, payoff ≥ 0 at expiry.

3. Calendar Arbitrage (Across Maturities)

For the same strike K, a longer-dated option must be worth at least the shorter-dated one (more time value, for American or forward-adjusted European calls):

$$C(K, T_2) \;\geq\; C(K, T_1) \quad \text{for } T_2 > T_1.$$

In terms of *total implied variance* $w(K, T) = \sigma^2_{\text{imp}}(K, T)\,T$, absence of calendar arbitrage requires w non-decreasing in T at fixed (forward) moneyness:

$$\frac{\partial w}{\partial T} \;\geq\; 0.$$

Calendar arbitrage: if $C(K, T_2) < C(K, T_1)$, buy the long-dated and sell the short-dated call (a long calendar spread) for a credit with a payoff that cannot be negative once the near leg expires.

4. Absolute Pricing Bounds

Model-free no-arbitrage bounds every option must satisfy (non-dividend, or with $S_0 \to S_0 e^{-qT}$):

$$\left(S_0 e^{-qT} - K e^{-rT}\right)^+ \;\leq\; C \;\leq\; S_0 e^{-qT},$$

$$\left(K e^{-rT} - S_0 e^{-qT}\right)^+ \;\leq\; P \;\leq\; K e^{-rT}.$$

- **Lower bound breach** ($C < (S_0 e^{-qT} - K e^{-rT})^+$): buy the call, short the replicating stock-minus-bond; lock the gap.
- **Upper bound breach** ($C > S_0 e^{-qT}$): sell the call, buy the stock; the call can never be worth more than the asset it delivers.

5. Chain-Scan Summary

An arbitrage-free chain simultaneously satisfies:

Constraint	Condition
Vertical (slope)	$0 \leq C(K_1) - C(K_2) \leq (K_2 - K_1)DF$
Convexity (butterfly)	$C(K_1) - 2C(K_2) + C(K_3) \geq 0$
Calendar	$C(K, T_2) \geq C(K, T_1),\ T_2 > T_1$
Absolute bounds	$(S_0 e^{-qT} - K e^{-rT})^+ \leq C \leq S_0 e^{-qT}$
Parity link	$C - P = S_0 e^{-qT} - K e^{-rT}$

A scanner flags any row that fails; each failure maps to a concrete spread trade (vertical, butterfly, calendar, or synthetic) that captures the discrepancy.

Practical Notes

- Test on *executable* bid/ask; mid-price "violations" often vanish after the spread.
- Convexity/calendar checks are best done on the implied-vol surface (total variance w), which normalizes for strike and maturity.
- Butterflies and calendars have small, capacity-limited edges; fees and pin risk decide viability.