Options: Box Spread

Long box / short box

Concept

A **box spread** combines a bull call spread and a bear put spread on the same two strikes $K_1 < K_2$ and expiry T. The four legs together have a payoff that is *constant*—always $K_2 - K_1$ at expiry, independent of the underlying. A box is therefore a synthetic zero-coupon bond: its fair price is the discounted strike difference. Mispricing lets you lend or borrow synthetically at an arbitrage rate.

Construction

The **long box** (buy the box) is:

- Bull call spread: buy call K_1, sell call K_2.
- Bear put spread: buy put K_2, sell put K_1.

The **short box** is the exact opposite (sell all four of the above legs).

Payoff (Always Constant)

Leg	$S_T \le K_1$	$K_1 < S_T < K_2$	$S_T \ge K_2$
Long call K_1	0	$S_T - K_1$	$S_T - K_1$
Short call K_2	0	0	$-(S_T - K_2)$
Long put K_2	$K_2 - S_T$	$K_2 - S_T$	0
Short put K_1	$-(K_1 - S_T)$	0	0
Total	$K_2 - K_1$	$K_2 - K_1$	$K_2 - K_1$

In every region the payoff is $K_2 - K_1$. The box is riskless.

Fair Value

Since the terminal value is certain, discount it:

$$\boxed{\text{Box}_{\text{fair}} \;=\; (K_2 - K_1)\, e^{-rT}}$$

Let the traded box cost be

$$B \;=\; \big(C_{K_1} - C_{K_2}\big) + \big(P_{K_2} - P_{K_1}\big),$$

the net premium paid for the long box.

Arbitrage Conditions

Long-box arbitrage (box too cheap)

If $B < (K_2 - K_1)e^{-rT}$, **buy the box**. You pay B now and receive the certain $K_2 - K_1$ at expiry. Risk-free profit (PV):

$$\pi \;=\; (K_2 - K_1)e^{-rT} - B \;>\; 0.$$

This is synthetic *lending* at an implied rate above r.

Short-box arbitrage (box too rich)

If $B > (K_2 - K_1)e^{-rT}$, **sell the box**. You collect B now and pay the fixed $K_2 - K_1$ at expiry. Risk-free profit (PV):

$$\pi \;=\; B - (K_2 - K_1)e^{-rT} \;>\; 0.$$

This is synthetic *borrowing* at an implied rate below r—often used to raise cash cheaply.

Implied Financing Rate

The box encodes an interest rate. Solving the fair-value relation:

$$r_{\text{box}} \;=\; \frac{1}{T}\,\ln\!\left(\frac{K_2 - K_1}{B}\right).$$

Compare to your funding rate r: lend via the box when $r_{\text{box}} > r$, borrow via the box when $r_{\text{box}} < r$.

Worked Example

$K_1 = 95$, $K_2 = 105$, $T = 0.5$, $r = 5\%$. Fair box $= 10\,e^{-0.05\times 0.5} = 10 \times 0.97531 = 9.753$. If the four legs cost $B = 9.60$, buy the box for a locked PV profit of $9.753 - 9.60 = \$0.153$ per share (\$15.30 per contract). Implied lending rate $r_{\text{box}} = \frac{1}{0.5}\ln(10/9.60) = 8.16\%$, well above 5%.

Practical Notes (and Warnings)

- **Use European options.** With American options, early assignment on a short leg can break the "riskless" payoff—the infamous risk in retail short boxes.
- Four-leg bid/ask spreads and commissions must be smaller than the gross edge.
- Pin risk at K_1 or K_2 (uncertain assignment at expiry) is a residual hazard.
- Cash-settled index options (e.g. SPX, European-style) are the safe venue for boxes.